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A new method selection approach for fuzzy group multicriteria decision making

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ABSTRACT

Fuzzy multicriteria decision making (MCDM) has been widely used in ranking a finite number of decision alternatives characterized by fuzzy assessments with respect to multiple criteria. In group decision settings, different fuzzy group MCDM methods often produce inconsistent ranking outcomes for the same problem. To address the ranking inconsistency problem in fuzzy group MCDM, this paper develops a new method selection approach for selecting a fuzzy group MCDM method that produces the most preferred group ranking outcome for a given problem. Based on two group averaging methods, three aggregation procedures and three defuzzification methods, 18 fuzzy group MCDM methods are developed as an illustration to solve the general fuzzy MCDM problem that requires cardinal ranking of the decision alternatives. The approach selects the group ranking outcome of a fuzzy MCDM method which has the highest consistency degree with its corresponding ranking outcomes of individual decision makers. An empirical study on the green bus fuel technology selection problem is used to illustrate how the approach works. The approach is applicable to large-scale group multicriteria decision problems where inconsistent ranking outcomes often exist between different fuzzy MCDM methods.

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1. Introduction

Multicriteria decision making (MCDM) has been widely used in evaluating, selecting or ranking a finite set of decision alternatives characterized by multiple and usually conflicting criteria. Bellman and Zadeh [1] first introduce fuzzy set theory as an effective methodology to deal with the inherent imprecision, vagueness and subjectiveness involved in the human decision making process. Numerous studies have since been conducted on the development of fuzzy MCDM methods and their applications to various MCDM problems involving imprecision and subjectiveness.

Fuzzy MCDM is concerned with evaluating a set of decision alternatives with respect to multiple criteria in a fuzzy environment where the criteria weights and alternatives' performance ratings are represented by fuzzy numbers [2–6]. Multiattribute value theory (MAVT) [7] has been widely used to solve fuzzy MCDM problems where a cardinal preference or ranking of decision alternatives is required. With simplicity in both concept and computation, MAVT-based MCDM methods are intuitively appealing to the decision makers in practical applications. In addition, these methods are the most appropriate quantitative tools for group decision support systems [8,9]. As such, this paper considers three widely used MAVT-based MCDM methods which are applicable

to large-scale decision problems where the evaluation or ranking outcomes produced by different methods are most likely to be significantly different.

MAVT-based fuzzy MCDM methods usually involve two steps. The first step aggregates the fuzzy weights and the fuzzy performance ratings as an overall fuzzy utility for assessing the overall performance of each decision alternative across all criteria. The second step compares the aggregated fuzzy utilities of the decision alternatives [2,3,10–14]. The aggregation in the first step can be made by an MAVT-based MCDM method. However, there is no best method for the general MCDM problem and different methods often produce inconsistent ranking outcomes for the same problem [15,16]. This is mainly due to the multiplicity and complexity of multicriteria decisions. To deal with the second step, a fuzzy number ranking method is usually applied.

Numerous fuzzy number ranking methods have been developed and there is no best method for the general fuzzy MCDM problem [17–20]. Most fuzzy number ranking methods suffer from various drawbacks such as (a) lack of sensitivity when comparing similar fuzzy numbers, (b) counterintuitive outcomes in certain circumstances, and (c) complex computational processes [20,21]. To address the fuzzy number ranking issue in fuzzy MCDM, defuzzification is widely used as an effective means for comparing fuzzy utilities of the decision alternatives [2,14,22]. Numerous defuzzification methods have been developed, and there is no best method. Often, each defuzzification method is used and examined in a specific decision context [14,20,21]. However, the relative

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Table 1
Linguistic terms for fuzzy weighting assessment.

Linguistic term	Not important (NI)	Somewhat important (SI)	Important (I)	Very important (VI)	Extremely important (EI)
Membership function	(0, 0, 3)	(0, 2.5, 5)	(3, 5, 7)	(5, 7.5, 10)	(7, 10, 10)

performance of these defuzzification methods in solving the general fuzzy MCDM problem is not clear. This makes the selection of a specific defuzzification method complex and difficult [22,23].

To effectively support fuzzy MCDM decisions, the selection of available MCDM and defuzzification methods with respect to their relative performance is thus an important issue. Due to the structural differences of available fuzzy MCDM methods, the ranking outcomes produced by these methods may not be consistent for a given decision problem. In fact, the empirical study presented in this paper shows that the ranking outcomes of the decision alternatives produced by different fuzzy MCDM methods are so different that the effectiveness of the methods used has to be examined in order to help select the most preferred ranking outcome. In a group decision making environment, this method selection issue is complicated by the fact that the decision makers often have different preferences of the decision alternatives.

In this paper, we develop a new method selection approach for selecting the most preferred ranking outcome among those produced by different fuzzy group MCDM methods. The most preferred ranking outcome is most acceptable to the decision makers as a whole, as it can best reflect their preferences in a specific decision setting. To illustrate the effectiveness of the approach, we consider 18 fuzzy group MCDM methods, which are developed from two group averaging methods, three MCDM methods and three defuzzification methods. These methods are intuitively appealing to the decision makers in practice due to their simplicity in both concept and computation. In practical applications, the approach can be applied to any number of group averaging, MCDM and defuzzification methods, as long as they are acceptable to the group of decision makers.

In subsequent sections, we first describe the general MAVT-based fuzzy group MCDM problem. Next, we present some commonly used methods for group averaging, MCDM, and defuzzification that can be combined to solve the fuzzy group MCDM problem. We then develop a new method selection approach for selecting among the different group ranking outcomes produced by available fuzzy group MCDM methods. Finally, we conduct an empirical study on the green bus fuel technology selection problem to demonstrate the effectiveness of the method selection approach.

2. The fuzzy group MCDM problem

The fuzzy group MCDM problem involves a finite set of m decision alternatives A_i ($i = 1, 2, \dots, m$), which are to be evaluated by a group of p decision makers DM_k ($k = 1, 2, \dots, p$) with respect to a set of n evaluation criteria C_j ($j = 1, 2, \dots, n$). These evaluation criteria are measurable quantitatively or assessable qualitatively, and are independent of each other. Assessments are to be made by each decision maker DM_k to determine (a) the fuzzy weight vector $W^k = (w^k_1, w^k_2, \dots, w^k_n)$ and (b) the fuzzy decision matrix $X^k = \{x^k_{ij}, i = 1, 2, \dots, m; j = 1, 2, \dots, n\}$.

The fuzzy weight vector W^k represents the fuzzy weights (relative importance) of the criteria C_j , which are given by the decision maker DM_k using a cardinal scale. The fuzzy decision matrix X^k

represents the fuzzy performance ratings (x_{ij}) of alternative A_i with respect to criteria C_j , which are either objectively measured (for quantitative criteria) or subjectively assessed by the decision maker DM_k (for qualitative criteria) using cardinal values.

In making subjective assessments of the fuzzy weight vector and the fuzzy decision matrix, the decision makers can use two types of judgment: comparative judgment and absolute judgment [5]. To facilitate comparative judgments, a pairwise comparison process is commonly used, as implemented in the analytic hierarchy process (AHP) [3,24]. In this paper, to illustrate how the fuzzy weight vector and the fuzzy decision matrix can be obtained by subjective assessments, we use absolute judgments as an example.

Subjective assessments are to be made by each decision maker DM_k to determine the fuzzy weight vector W^k and the fuzzy decision matrix X^k , using absolute judgments with the linguistic terms given in Tables 1 and 2. In practical applications, triangular fuzzy numbers are most widely used to represent the approximate value range of the linguistic term [5]. The popular use of triangular fuzzy numbers is mainly attributed to their simplicity in both concept and computation. Theoretically, the merits of using triangular fuzzy numbers in fuzzy modeling have been well justified [25]. With the simplest form of the membership function, triangular fuzzy numbers constitute an immediate solution to the optimization problems in fuzzy modeling. In a triangular fuzzy number denoted as (a_1, a_2, a_3) where $a_1 < a_2 < a_3$, a_2 is the most possible assessment value, and a_1 and a_3 are the lower and upper bounds respectively for reflecting the fuzziness of the assessment. Table 1 shows a typical set of linguistic terms, together with their corresponding membership functions, for the linguistic variable “importance”, which can be used to assess the criteria weight. To assess the performance rating of alternatives, we need another set of linguistic terms for the linguistic variable “performance”, as given in Table 2.

The intervals of the membership functions used in Tables 1 and 2 are suggested in [2] for a linguistic variable with a set of five linguistic terms and a value range between 0 and 10. To reflect the fact that the decision makers may have different perceptions of these linguistic terms in practical applications, each decision maker has the option of defining the membership functions for these linguistic terms as the assessment result. If the decision maker has no personal preference in using these linguistic terms, the membership functions defined in Tables 1 and 2 can be used as default values. This setting is implemented in the empirical study.

With the use of the linguistic terms in Tables 1 and 2 for assessing the criteria weights and the performance ratings of the alternatives, a fuzzy weight vector W^k and a fuzzy decision matrix X^k are obtained for each of p decision makers DM_k . Given p fuzzy weight vectors W^k and p fuzzy decision matrices X^k , the objective of the evaluation problem is to rank all the alternatives by giving each of them an overall preference value with respect to all the criteria.

3. Development of fuzzy group MCDM methods

The solution procedure for the fuzzy group MCDM problem typically involves three key phases: group averaging, MCDM

Table 2
Linguistic terms for fuzzy performance rating assessment.

Linguistic term	Very poor (VP)	Poor (P)	Fair (F)	Good (G)	Very good (VG)
Membership function	(0, 0, 3)	(0, 2.5, 5)	(3, 5, 7)	(5, 7.5, 10)	(7, 10, 10)

aggregation, and defuzzification. The group averaging phase is used to aggregate or average individual fuzzy weight vectors and individual fuzzy decision matrices into a group fuzzy weight vector and a group fuzzy decision matrix. The MCDM aggregation phase is used to synthesize the fuzzy group weight vector and the fuzzy group decision matrix in order to obtain an overall fuzzy preference value for each alternative. The defuzzification phase is used to obtain a crisp preference value for each alternative, on which the ranking of all the alternatives can be based. With the use of triangular fuzzy numbers, the arithmetic operations on fuzzy numbers are based on interval arithmetic [26]. Commonly used methods for the three phases of a typical fuzzy group MCDM method are presented in the following sections.

3.1. The group averaging phase

Many operators have been proposed for aggregating information such as arithmetic averaging, weighted arithmetic averaging, geometric averaging, and weighted geometric averaging [27]. Arithmetic mean and geometric mean are among the most used methods for averaging individual assessments of decision makers. Once all the fuzzy numbers are obtained, the arithmetic and geometric means can be performed to obtain the average evaluation rating of each criterion and the average performance of each alternative.

The cardinal values given in the fuzzy weight vector $W^k = (w^k_1, w^k_2, \dots, w^k_n)$ and the fuzzy decision matrix $X^k = \{x^k_{ij}, i = 1, 2, \dots, m; j = 1, 2, \dots, n\}$ represent the absolute preferences of the decision maker DM_k . These individual fuzzy weight vectors and fuzzy decision matrices are averaged to represent the group fuzzy weight vector W and group fuzzy decision matrix X . As such, the group fuzzy weight vector is given by

$$W = (w_1, w_2, \dots, w_n) \tag{1}$$

where $w_j = \sum_{k=1}^p w_j^k / p$ if the arithmetic mean method is used or $w_j = \prod_{k=1}^p (w_j^k)^{1/p}$ if the geometric mean method is used; $j = 1, 2, \dots, n$. If the decision makers DM_k have different importance values (i.e. carry different weights) λ^k in the decision making process, $w_j = \sum_{k=1}^p w_j^k \lambda^k / \sum_{q=1}^p \lambda^q$ if the arithmetic mean method is used or $w_j = \prod_{k=1}^p (w_j^k)^{\lambda^k / \sum_{q=1}^p \lambda^q}$ if the geometric mean method is used; $j = 1, 2, \dots, n$.

The group fuzzy decision matrix X is given by

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \tag{2}$$

where $x_{ij} = \sum_{k=1}^p x_{ij}^k / p$ if the arithmetic mean method is used, or $x_{ij} = \prod_{k=1}^p (x_{ij}^k)^{1/p}$ if the geometric mean method is used; $i = 1, 2, \dots, m; j = 1, 2, \dots, n$. If the decision makers DM_k have different importance values λ^k , $x_{ij} = \sum_{k=1}^p x_{ij}^k \lambda^k / \sum_{q=1}^p \lambda^q$ if the arithmetic mean method is used, or $x_{ij} = \prod_{k=1}^p (x_{ij}^k)^{\lambda^k / \sum_{q=1}^p \lambda^q}$ if the geometric mean method is used; $i = 1, 2, \dots, m; j = 1, 2, \dots, n$.

For easy illustration of the new fuzzy group MCDM method selection approach, in this paper we assume that the opinions of all p decision makers on the criteria weights and the alternatives' performance ratings are weighted equally.

3.2. The MCDM aggregation phase

Three widely used MAVT-based MCDM methods described below can be used for the aggregation phase [28].

3.2.1. The simple additive weighting (SAW) method

The SAW method, also known as the weighted sum method, is probably the best known and most widely used MCDM method [29]. The basic logic of the SAW method is to obtain a weighted sum of the performance ratings of each alternative over all criteria. The SAW method normally requires normalizing the fuzzy decision matrix (X) to allow a comparable scale for all ratings in X by

$$r_{ij} = \begin{cases} x_{ij}/\max_i x_{ij} & \text{if } j \text{ is a benefit criterion} \\ \min_i x_{ij}/x_{ij} & \text{if } j \text{ is a cost criterion} \end{cases}; \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n. \tag{3}$$

where r_{ij} ($0 \leq r_{ij} \leq 1$) is defined as the normalized fuzzy performance rating of alternative A_i on criteria C_j . This normalization process transforms all the ratings in a linear way, so that the relative order of magnitude of the ratings remains equal. The overall fuzzy preference value (V_i) of each alternative is obtained by:

$$V_i = \sum_{j=1}^n w_j r_{ij}; \quad i = 1, 2, \dots, m. \tag{4}$$

The greater the value (V_i), the more preferred the alternative (A_i).

3.2.2. The weighted product (WP) method

The WP method uses multiplication for connecting criteria ratings, each of which is raised to the power of the corresponding criteria weight [30,31]. This multiplication process has the same effect as the normalization process for handling different measurement units. The fuzzy preference value of each alternative is given by

$$S_i = \prod_{j=1}^n x_{ij}^{w_j}; \quad i = 1, 2, \dots, m. \tag{5}$$

where $\sum_{j=1}^n w_j = 1$. w_j is a positive power for benefit criteria and a negative power for cost criteria. Due to the exponentiation property, if a criterion has fractional ratings, all ratings with respect to that criterion are multiplied by 10^q ($q \geq 1$) to make all ratings greater than one. In this study, for easy comparison with the preference values generated by the other two methods, the overall fuzzy preference value (V_i) of each alternative is given by:

$$V_i = \frac{\prod_{j=1}^n x_{ij}^{w_j}}{\prod_{j=1}^n (x * j)^{w_j}}, \quad i = 1, 2, \dots, m. \tag{6}$$

where $x * j = \max_i x_{ij}$ and $0 \leq V_i \leq 1$. The greater the value (V_i), the more preferred the alternative (A_i).

3.2.3. The technique for order preference by similarity to ideal solution (TOPSIS)

TOPSIS is based on the concept that the most preferred alternative should not only have the shortest distance from the positive ideal solution, but also have the longest distance from the negative ideal solution [29,32]. This concept has been widely used in various MCDM models for solving practical decision problems [33–38]. This

is due to (a) its simplicity and comprehensibility in concept, (b) its computational efficiency, and (c) its ability to measure the relative performance of the decision alternatives in a simple mathematical form.

TOPSIS normally requires normalizing the fuzzy performance ratings of alternative A_i on criteria C_j by:

$$r_{ij} = x_{ij} / \sqrt{\sum_{i=1}^m x_{ij}^2}; \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n. \quad (7)$$

The positive ideal solution A^+ and the negative ideal solution A^- are determined based on the weighted normalized fuzzy ratings (y_{ij}) by

$$y_{ij} = w_j r_{ij}; \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n. \quad (8)$$

$$A^+ = (y_1^+, y_2^+, \dots, y_n^+); \quad A^- = (y_1^-, y_2^-, \dots, y_n^-) \quad (9)$$

where $y_j^+ = \begin{cases} \max_i y_{ij}, & \text{if } j \text{ is a benefit criterion} \\ \min_i y_{ij}, & \text{if } j \text{ is a cost criterion} \end{cases}; \quad y_j^- = \begin{cases} \min_i y_{ij}, & \text{if } j \text{ is a benefit criterion} \\ \max_i y_{ij}, & \text{if } j \text{ is a cost criterion} \end{cases};$
 $j = 1, 2, \dots, n. \quad (10)$

The distance between A_i and A^+ , and the distance between A_i and A^- are calculated respectively by:

$$D_i^+ = \sqrt{\sum_{j=1}^n (y^+ - y_{ij})^2};$$

$$D_i^- = \sqrt{\sum_{j=1}^n (y_{ij} - y^-)^2}; \quad i = 1, 2, \dots, m. \quad (11)$$

The overall fuzzy preference value (V_i) of each alternative is given by:

$$V_i = \frac{D_i^-}{D_i^+ + D_i^-}; \quad i = 1, 2, \dots, m. \quad (12)$$

The greater the value (V_i), the more preferred the alternative (A_i).

3.3. The defuzzification phase

Defuzzification is the process to convert a fuzzy number into a non-fuzzy (crisp) value. As an illustration of the method selection approach, three simple and commonly used defuzzification methods for triangular fuzzy numbers are used in this paper, including center-of-area (COA), graded mean integration (GMI), and metric distance (MD). These methods differ in their way of weighting the most possible assessment value.

The COA method is the most popular and commonly used method to defuzzify a triangular fuzzy number [39–44]. Given an overall fuzzy preference value $V_i = (a_i, b_i, c_i)$, the defuzzification value by the COA method is obtained by

$$R(V_i) = \frac{(c_i - a_i) + (b_i - a_i)}{3} + a_i \quad \text{or} \quad R(V_i) = \frac{a_i + b_i + c_i}{3} \quad (13)$$

The GMI method defuzzifies a triangular fuzzy number as [37,45,46]:

$$R(V_i) = \frac{a_i + 4b_i + c_i}{6} \quad (14)$$

Table 3
Eighteen group ranking methods and their corresponding reference code.

Group averaging	MCDM method	Defuzzification	Code
Arithmetic mean	SAW	COA	A-S-C
Arithmetic mean	SAW	GMI	A-S-G
Arithmetic mean	SAW	MD	A-S-M
Arithmetic mean	WP	COA	A-W-C
Arithmetic mean	WP	GMI	A-W-G
Arithmetic mean	WP	MD	A-W-M
Arithmetic mean	TOPSIS	COA	A-T-C
Arithmetic mean	TOPSIS	GMI	A-T-G
Arithmetic mean	TOPSIS	MD	A-T-M
Geometric mean	SAW	COA	G-S-C
Geometric mean	SAW	GMI	G-S-G
Geometric mean	SAW	MD	G-S-M
Geometric mean	WP	COA	G-W-C
Geometric mean	WP	GMI	G-W-G
Geometric mean	WP	MD	G-W-M
Geometric mean	TOPSIS	COA	G-T-C
Geometric mean	TOPSIS	GMI	G-T-G
Geometric mean	TOPSIS	MD	G-T-M

The defuzzification value by the MD method [47–51] is given by:

$$R(V_i) = \frac{a_i + 2b_i + c_i}{4} \quad (15)$$

3.4. Development of fuzzy group MCDM methods

Combining the two methods for averaging individual judgments of the decision makers (arithmetic mean and geometric mean) with three aggregation procedures (SAW, WP, and TOPSIS) and three defuzzification methods (COA, GMI, and MD) will result in 18 different fuzzy group MCDM methods. Table 3 shows these 18 methods, each is associated with a code for easy reference. These methods can be used to solve the general fuzzy group MCDM problem that requires cardinal ranking of all the alternatives.

4. A new approach to selecting the most preferred group decision outcome

A number of methods are often available for solving a fuzzy MCDM problem. Due to their structural differences, different fuzzy MCDM methods often produce inconsistent ranking outcomes for the same problem. Despite significant developments in MCDM method selection research, the validation of ranking outcomes remains an open issue [52,53]. This is mainly due to the fact that the “true” cardinal ranking of alternatives is unknown [16]. In addition, there is no such thing as the “right answer” as the concept of an optimum does not exist in a multicriteria framework [54]. This implies that the “true” ranking of alternatives is not known or cannot be obtained in a universally accepted way. To address this issue for the 18 available fuzzy MCDM methods illustrated in this paper, we develop a new approach for selecting the most preferred ranking outcome among all feasible outcomes produced by available fuzzy MCDM methods in the context of group decision making.

In group decision making, the stakeholders or decision makers often have different views of the criteria weights (i.e. the weight vector) and the alternatives’ performance ratings (i.e. the decision matrix). To reach a compromised solution, the assessment values given by individual decision makers for the weight vector and the decision matrix are often averaged as the group values. As such, the individual ranking outcomes based on the values given by individual decision makers may not be consistent with the final group ranking outcome derived from the averaged group values. In addition, different fuzzy MCDM methods often produce different final group ranking outcomes for the same group fuzzy weight vector W and group fuzzy decision matrix X , as suggested by existing studies

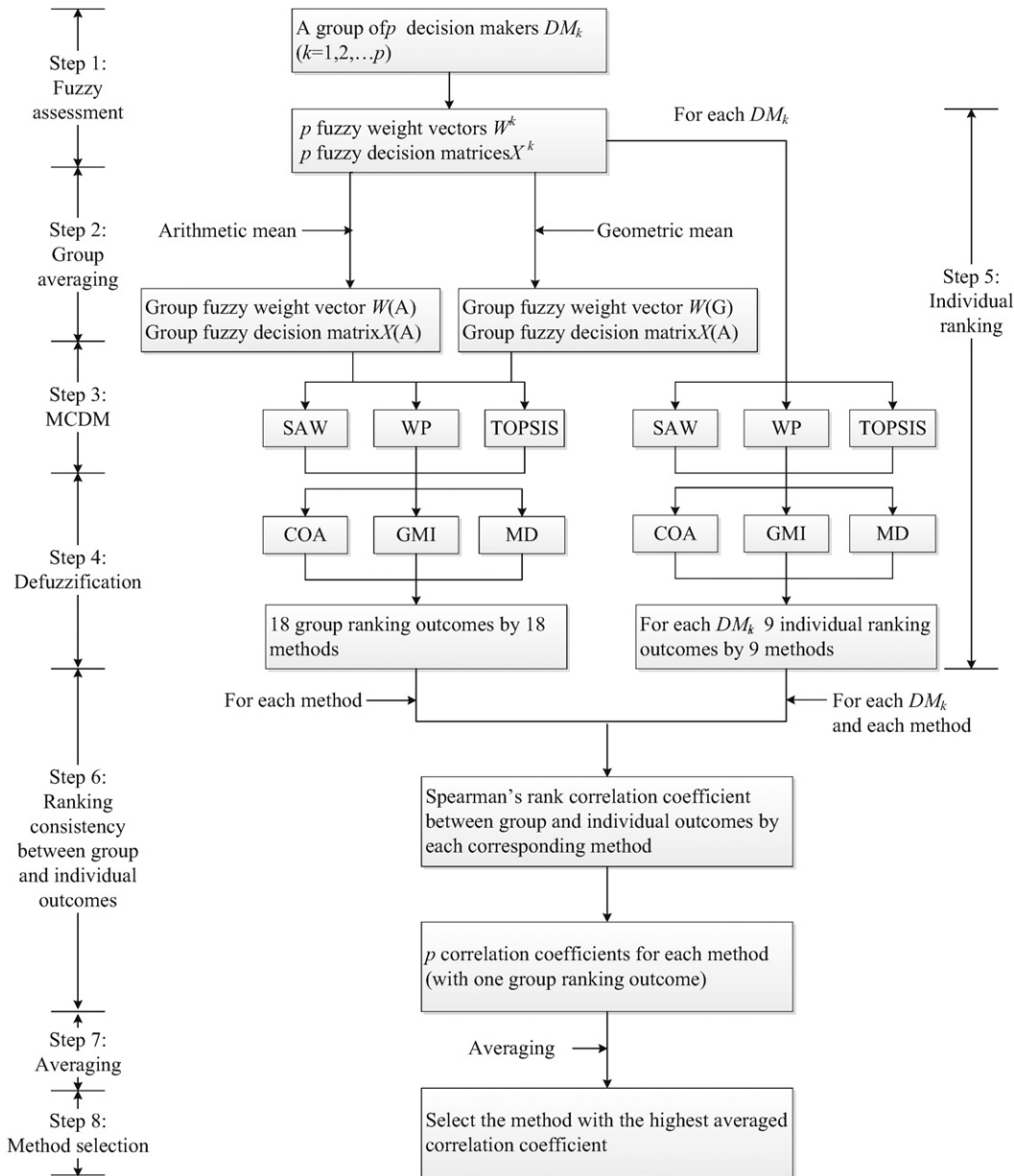


Fig. 1. The fuzzy group MCDM method selection approach.

[16,55] and evidenced in the empirical study of this paper. Among all the feasible group ranking outcomes, individual decision makers would prefer the group ranking outcome which is most consistent with their own ranking outcome. That is, the most preferred group ranking outcome is the one that is most consistent with individual rankings made by respective decision makers using a particular MCDM method. This is the notion on which the new method selection approach is based.

With m decision alternatives A_i ($i = 1, 2, \dots, m$) and p decision makers DM_k ($k = 1, 2, \dots, p$) using a fuzzy group MCDM method, one group ranking outcome V_i and p individual ranking outcomes V_i^k will be produced. The consistency degree or the correlation between V_i and each V_i^k using the corresponding methods can be measured by Spearman's rank correlation coefficient, resulting in p correlation coefficients. For example, the individual ranking outcome V_i^1 by DM_1 using SAW and COA is to be compared with the group ranking outcome V_i using the A-S-C method. With one decision maker DM_1 , this produces one correlation coefficient for one fuzzy group MCDM method (i.e. the A-S-C method). With p decision makers,

p correlation coefficients will be produced for the A-S-C method. When applying this process to other fuzzy group MCDM methods individually, p correlation coefficients will be produced for each method.

The overall consistency degree of each fuzzy group MCDM method can be obtained by averaging the p correlation coefficients. The method selection approach will select the group ranking outcome of a fuzzy MCDM method which has the highest consistency degree, as compared to that of other methods. This implies that the method selected is the most preferred one, as the ranking outcome produced is most acceptable by the decision makers as a whole.

Based on the fuzzy group MCDM problem setting described above, the new method selection approach is illustrated in Fig. 1 and summarized below. Obtain the fuzzy weight vectors and the fuzzy decision matrices assessed by p individual decision makers (DM_k , $k = 1, 2, \dots, p$) using the linguistic terms defined in Tables 1 and 2. Average individual fuzzy weight vectors and individual fuzzy decision matrices into two group fuzzy weight vectors and two group fuzzy decision matrices by arithmetic mean and

Table 4
Individual fuzzy weight vectors of 12 decision makers.

	C_1	C_2	C_3	C_4	C_5	C_6
DM_1	(5, 7.5, 10)	(3, 5, 7)	(5, 7.5, 10)	(3, 5, 7)	(7, 10, 10)	(3, 5, 7)
DM_2	(5, 7.5, 10)	(5, 7.5, 10)	(5, 7.5, 10)	(5, 7.5, 10)	(5, 7.5, 10)	(5, 7.5, 10)
DM_3	(6, 7.5, 9)	(1, 2.5, 4)	(3.5, 5, 6.5)	(3.5, 5, 6.5)	(3.5, 5, 6.5)	(3.5, 5, 6.5)
DM_4	(5, 7.5, 10)	(7, 10, 10)	(7, 10, 10)	(5, 7.5, 10)	(3, 5, 7)	(3, 5, 7)
DM_5	(5, 7.5, 10)	(5, 7.5, 10)	(7, 10, 10)	(7, 10, 10)	(5, 7.5, 10)	(3, 5, 7)
DM_6	(5, 7.5, 10)	(5, 7.5, 10)	(7, 10, 10)	(5, 7.5, 10)	(5, 7.5, 10)	(3, 5, 7)
DM_7	(5, 7.5, 10)	(3, 5, 7)	(3, 5, 7)	(5, 7.5, 10)	(5, 7.5, 10)	(3, 5, 7)
DM_8	(5, 7.5, 10)	(5, 7.5, 10)	(7, 10, 10)	(5, 7.5, 10)	(7, 10, 10)	(7, 10, 10)
DM_9	(7, 10, 10)	(5, 7.5, 10)	(7, 10, 10)	(5, 7.5, 10)	(5, 7.5, 10)	(3, 5, 7)
DM_{10}	(3, 5, 7)	(5, 7.5, 10)	(3, 5, 7)	(5, 7.5, 10)	(5, 7.5, 10)	(3, 5, 7)
DM_{11}	(5, 7.5, 10)	(5, 7.5, 10)	(5, 7.5, 10)	(5, 7.5, 10)	(7, 10, 10)	(3, 5, 7)
DM_{12}	(5, 7.5, 10)	(7, 10, 10)	(5, 7.5, 10)	(5, 7.5, 10)	(3, 5, 7)	(3, 5, 7)

Table 5
Individual fuzzy decision matrix of DM_1 .

	A_1	A_2	A_3	A_4	A_5	A_6
C_1	(5, 7.5, 10)	(5, 7.5, 10)	(3, 5, 7)	(0, 2.5, 5)	(5, 7.5, 10)	(5, 7.5, 10)
C_2	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(5, 7.5, 10)	(5, 7.5, 10)
C_3	(5, 7.5, 10)	(5, 7.5, 10)	(0, 2.5, 5)	(0, 2.5, 5)	(5, 7.5, 10)	(7, 10, 10)
C_4	(3, 5, 7)	(3, 5, 7)	(5, 7.5, 10)	(0, 2.5, 5)	(0, 2.5, 5)	(0, 2.5, 5)
C_5	(5, 7.5, 10)	(5, 7.5, 10)	(0, 2.5, 5)	(0, 2.5, 5)	(5, 7.5, 10)	(5, 7.5, 10)
C_6	(5, 7.5, 10)	(3, 5, 7)	(5, 7.5, 10)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)

geometric mean respectively, as expressed in Eqs. (1) and (2). Aggregate each group fuzzy weight vector and group fuzzy decision matrix by three MCDM methods (SAW, WP, and TOPSIS) respectively to obtain six sets of group fuzzy preference values. Use three defuzzification methods (COA, GMI, and MD) on the six sets of group fuzzy preference values to obtain 18 group ranking outcomes by the 18 methods. For each of p decision makers, apply three MCDM methods and three defuzzification methods respectively to each set of individual fuzzy weight vectors and fuzzy decision matrices obtained at Step 1 to obtain nine individual ranking outcomes. For each of p decision makers, determine the consistency degree between 18 group ranking outcomes and nine individual ranking outcomes produced by the corresponding MCDM and defuzzification methods, using Spearman's rank correlation coefficient. For each of 18 group ranking outcomes by the 18 methods, average the p correlation coefficients obtained at Step 6. Select the fuzzy group MCDM method that produces the highest averaged correlation coefficient at Step 7.

5. Empirical study on green bus fuel technology selection

To illustrate the inconsistent ranking outcomes produced by different fuzzy MCDM methods, we present a green bus fuel

Table 6
Group fuzzy weight vectors and group fuzzy decision matrices.

	Criteria weight		A_1		A_2		A_3
	Arithmetic	Geometric	Arithmetic	Geometric	Arithmetic	Geometric	Arithmetic
C_1	(0.51, 0.75, 0.97)	(0.50, 0.74, 0.96)	(0.42, 0.65, 0.85)	(0, 0.61, 0.83)	(0.49, 0.73, 0.92)	(0.47, 0.71, 0.91)	(0.29, 0.52, 0.75)
C_2	(0.47, 0.71, 0.9)	(0.42, 0.67, 0.87)	(0.39, 0.635, 0.89)	(0, 0.58, 0.82)	(0.33, 0.57, 0.76)	(0, 0.53, 0.74)	(0.34, 0.5, 0.79)
C_3	(0.54, 0.8, 0.92)	(0.51, 0.77, 0.91)	(0.45, 0.69, 0.88)	(0.41, 0.65, 0.85)	(0.43, 0.67, 0.88)	(0, 0.64, 0.86)	(0.35, 0.58, 0.77)
C_4	(0.49, 0.71, 0.93)	(0.48, 0.68, 0.91)	(0.43, 0.65, 0.87)	(0.41, 0.63, 0.85)	(0.32, 0.54, 0.77)	(0, 0.5, 0.74)	(0.31, 0.55, 0.77)
C_5	(0.5, 0.75, 0.92)	(0.48, 0.73, 0.91)	(0.3, 0.52, 0.74)	(0, 0.48, 0.72)	(0.43, 0.65, 0.83)	(0, 0.59, 0.81)	(0.28, 0.53, 0.71)
C_6	(0.35, 0.53, 0.71)	(0.34, 0.49, 0.7)	(0.34, 0.56, 0.76)	(0, 0.52, 0.74)	(0.32, 0.52, 0.73)	(0, 0.5, 0.72)	(0.29, 0.53, 0.73)
	A_3		A_4		A_5		A_6
	Geometric	Arithmetic	Geometric	Arithmetic	Geometric	Arithmetic	Geometric
C_1	(0, 0.47, 0.72)	(0.2, 0.48, 0.71)	(0, 0.43, 0.68)	(0.37, 0.61, 0.81)	(0, 0.56, 0.79)	(0.37, 0.61, 0.84)	(0, 0.57, 0.81)
C_2	(0, 0.53, 0.77)	(0.38, 0.61, 0.8)	(0.4, 0.58, 0.78)	(0.43, 0.71, 0.9)	(0.45, 0.69, 0.88)	(0.49, 0.77, 0.95)	(0.51, 0.75, 0.94)
C_3	(0, 0.52, 0.74)	(0.41, 0.69, 0.85)	(0, 0.64, 0.83)	(0.38, 0.65, 0.85)	(0.37, 0.61, 0.82)	(0.32, 0.54, 0.74)	(0, 0.49, 0.71)
C_4	(0, 0.5, 0.74)	(0.35, 0.63, 0.86)	(0, 0.59, 0.83)	(0.37, 0.65, 0.88)	(0, 0.62, 0.86)	(0.39, 0.67, 0.9)	(0, 0.64, 0.88)
C_5	(0, 0.45, 0.68)	(0.24, 0.48, 0.69)	(0, 0.41, 0.66)	(0.18, 0.4, 0.62)	(0, 0, 0.6)	(0.28, 0.5, 0.7)	(0, 0.45, 0.67)
C_6	(0, 0.47, 0.7)	(0.25, 0.5, 0.7)	(0, 0.46, 0.68)	(0.33, 0.59, 0.8)	(0.35, 0.58, 0.78)	(0.39, 0.67, 0.85)	(0, 0.63, 0.83)

technology selection problem in Taiwan. To show how the new method selection approach can be used to select the most preferred group ranking outcome for the problem, we apply the 18 fuzzy group MCDM methods presented in Table 3.

The objective of the green bus fuel technology selection problem is to select a new green fuel technology for buses in Taiwan. The six alternatives considered are natural gas (A_1), liquefied petroleum gas (A_2), methanol and ethanol (A_3), biodiesel (A_4), hybrid electric (A_5), and fuel cell (A_6). Based on comprehensive discussions with the experts in relevant public and private sectors in Taiwan, a set of six selection criteria is determined. We briefly discuss these criteria below.

- (a) Supply (C_1): the degree of the long term availability of the fuel.
- (b) Emission (C_2): the average emission of a given pollutant occurring as a result of using the fuel.
- (c) Technology (C_3): the capability to convert existing buses into using the fuel.
- (d) Safety (C_4): the state of being safe for consumers and the society after using the fuel.
- (e) Cost (C_5): the overall cost to convert buses into using the alternative fuels.
- (f) Consumer preference (C_6): the consumer's willingness and preference of using the fuel.

A survey with structured questionnaires was conducted to ask the experts in Taiwan to (a) weight the six selection criteria, and (b) assess the performance rating of six green bus fuel technology alternatives with respect to each selection criterion using the linguistic terms defined in Tables 1 and 2 independently. For their fuzzy assessment results, the experts had the option of using the

Table 7
Group ranking outcome of 18 methods.

Method	A ₁ Value (ranking)	A ₂ Value (ranking)	A ₃ Value (ranking)	A ₄ Value (ranking)	A ₅ Value (ranking)	A ₆ Value (ranking)
A-S-C	2.722 (1)	2.709 (3)	2.412 (6)	2.475 (5)	2.616 (4)	2.710 (2)
A-S-G	2.672 (2)	2.669 (3)	2.365 (6)	2.445 (5)	2.580 (4)	2.674 (1)
A-S-M	2.697 (1)	2.689 (3)	2.388 (6)	2.460 (5)	2.598 (4)	2.692 (2)
A-W-C	0.188 (1)	0.181 (3)	0.114 (6)	0.124 (5)	0.155 (4)	0.183 (2)
A-W-G	0.157 (1)	0.154 (3)	0.095 (5)	0.106 (6)	0.130 (4)	0.155 (2)
A-W-M	0.172 (1)	0.167 (3)	0.105 (6)	0.115 (5)	0.142 (4)	0.169 (2)
A-T-C	0.992 (2)	1 (1)	0.446 (6)	0.534 (5)	0.743 (4)	0.918 (3)
A-T-G	0.966 (2)	1 (1)	0.458 (6)	0.567 (5)	0.752 (4)	0.919 (3)
A-T-M	0.978 (2)	1 (1)	0.452 (6)	0.551 (5)	0.748 (4)	0.919 (3)
G-S-C	2.208 (2)	2.201 (3)	1.943 (6)	2.022 (5)	2.070 (4)	2.210 (1)
G-S-G	2.299 (2)	2.298 (3)	1.979 (6)	2.080 (4)	2.067 (5)	2.299 (1)
G-S-M	2.254 (2)	2.250 (3)	1.961 (6)	2.051 (5)	2.069 (4)	2.254 (1)
G-W-C	0.141 (1)	0.137 (3)	0.079 (6)	0.092 (4)	0.091 (5)	0.138 (2)
G-W-G	0.125 (1)	0.122 (2)	0.067 (5)	0.079 (4)	0.046 (6)	0.121 (3)
G-W-M	0.133 (1)	0.129 (2)	0.073 (5)	0.085 (4)	0.068 (6)	0.129 (3)
G-T-C	1 (1)	0.995 (2)	0.513 (6)	0.648 (5)	0.805 (4)	0.932 (3)
G-T-G	0.987 (2)	1 (1)	0.657 (5)	0.713 (4)	0.576 (6)	0.930 (3)
G-T-M	0.993 (2)	1 (1)	0.598 (6)	0.686 (4)	0.681 (5)	0.931 (3)

Table 8
Ranking outcome of DM₁.

Method	A ₁ Value (ranking)	A ₂ Value (ranking)	A ₃ Value (ranking)	A ₄ Value (ranking)	A ₅ Value (ranking)	A ₆ Value (ranking)
DM ₁ -S-C	2.653 (2)	2.562 (4)	1.723 (5)	1.278 (6)	2.609 (3)	2.705 (1)
DM ₁ -S-G	2.545 (2)	2.468 (4)	1.580 (5)	1.170 (6)	2.523 (3)	2.665 (1)
DM ₁ -S-M	2.598 (2)	2.515 (5)	1.651 (4)	1.223 (6)	2.566 (3)	2.685 (1)
DM ₁ -T-C	0.776 (1)	0.758 (2)	0.280 (5)	0.000 (6)	0.746 (4)	0.796 (3)
DM ₁ -T-G	0.767 (1)	0.754 (3)	0.254 (5)	0.000 (6)	0.752 (4)	0.827 (2)
DM ₁ -T-M	0.771 (1)	0.756 (3)	0.267 (5)	0.000 (6)	0.749 (4)	0.811 (2)
DM ₁ -W-C	0.313 (2)	0.257 (3)	0.061 (5)	0.020 (6)	0.227 (4)	0.247 (1)
DM ₁ -W-G	0.291 (2)	0.250 (4)	0.046 (5)	0.016 (6)	0.239 (3)	0.279 (1)
DM ₁ -W-M	0.301 (2)	0.253 (4)	0.054 (5)	0.018 (6)	0.233 (3)	0.263 (1)

Table 9
Consistency degree between 18 group ranking outcomes and individual ranking outcomes.

	A-S-C	A-S-G	A-S-M	A-W-C	A-W-G	A-W-M	A-T-C	A-T-G	A-T-M
DM ₁	0.829	0.886	0.657	0.886	0.943	0.943	0.714	0.543	0.543
DM ₂	0.812	0.899	0.812	0.600	0.600	0.600	0.899	0.899	0.899
DM ₃	0.200	0.029	0.200	-0.314	-0.314	-0.314	0.486	0.086	0.086
DM ₄	0.794	0.618	0.794	0.754	0.580	0.754	0.870	0.870	0.870
DM ₅	0.600	0.771	0.600	0.600	0.600	0.600	0.657	0.657	0.657
DM ₆	0.319	0.058	0.319	0.319	0.319	0.319	0.580	0.580	0.580
DM ₇	0.371	0.314	0.257	-0.029	-0.029	-0.029	-0.257	-0.257	-0.486
DM ₈	0.371	0.257	0.371	0.543	0.543	0.543	-0.029	0.143	0.143
DM ₉	-0.600	-0.486	-0.600	-0.600	-0.600	-0.600	-0.143	-0.143	-0.086
DM ₁₀	0.600	0.657	0.600	0.600	0.600	0.600	0.086	0.086	0.029
DM ₁₁	-0.543	-0.314	-0.543	-0.543	-0.543	-0.543	-0.657	-0.657	-0.657
DM ₁₂	0.486	-0.543	-0.486	-0.486	-0.486	-0.486	-0.257	-0.257	-0.257
Average	0.353	0.262	0.248	0.194	0.184	0.199	0.246	0.213	0.193
Ranking	1	3	5	11	14	10	7	9	12
	G-S-C	G-S-G	G-S-M	G-W-C	G-W-G	G-W-M	G-T-C	G-T-G	G-T-M
DM ₁	0.886	0.886	0.714	0.771	0.714	0.714	0.771	0.257	0.371
DM ₂	0.899	0.899	0.899	0.486	0.371	0.371	0.812	0.725	0.841
DM ₃	0.029	0.029	0.029	-0.143	0.314	0.314	0.429	0.486	0.257
DM ₄	0.618	0.618	0.618	0.551	0.116	0.406	0.928	0.580	0.725
DM ₅	0.771	0.771	0.771	0.429	0.257	0.257	0.543	0.371	0.486
DM ₆	0.058	0.058	0.058	0.203	0.406	0.406	0.580	0.406	0.464
DM ₇	0.429	0.314	0.314	-0.257	-0.600	-0.600	-0.143	-0.600	-0.600
DM ₈	0.257	0.257	0.257	0.486	0.200	0.200	0.086	-0.257	0.029
DM ₉	-0.486	-0.486	-0.486	-0.371	-0.143	-0.143	-0.314	0.314	0.200
DM ₁₀	0.657	0.657	0.657	0.543	0.143	0.143	0.314	-0.086	0.086
DM ₁₁	-0.314	-0.314	-0.314	-0.371	-0.486	-0.486	-0.714	-0.429	-0.486
DM ₁₂	-0.543	-0.543	-0.543	-0.314	0.143	0.143	-0.371	0.086	-0.143
Average	0.272	0.262	0.248	0.168	0.120	0.144	0.243	0.154	0.186
Ranking	2	3	6	15	18	17	8	16	13

membership functions defined in Tables 1 and 2 as default values or specifying the membership function for each linguistic term.

A group of 12 experts were selected to act as 12 decision makers (DM_i , $i = 1, 2, \dots, 12$) for the evaluation problem, including 4 practitioners from the bus operating companies, 4 government officials from the public transport sector, and 4 academic researchers. 11 experts used the default membership functions defined for the linguistic terms in Tables 1 and 2, and one expert (DM_3) specified the membership functions as $\{(0, 0, 1), (1, 2.5, 4), (3.5, 5, 6.5), (6, 7.5, 9), (9, 10, 10)\}$. This survey resulted in 12 individual fuzzy weight vectors and 12 individual fuzzy decision matrices. Table 4 shows the individual fuzzy weight vectors assessed by the 12 decision makers. As an illustration, Table 5 shows the individual fuzzy decision matrix of DM_1 .

Two group fuzzy weight vectors and two fuzzy decision matrices are obtained by applying the arithmetic mean and geometric mean methods respectively, as shown in Table 6. By applying three MCDM methods and three defuzzification methods to the two group fuzzy weight vectors and the two fuzzy decision matrices in Table 6, 18 group ranking outcomes are obtained. As shown in Table 7, these 18 group ranking outcomes are not consistent. Depending on the method selected, natural gas (A_1), liquefied petroleum gas (A_2) or fuel cell (A_6) can be the solution for the problem.

To select a green bus fuel technology that is most preferred by the decision makers as a whole, we apply the method selection approach developed in this paper. For easy comparison with the corresponding group ranking outcome, the method used by individual decision makers (DM_i , $i = 1, 2, \dots, 12$) with three different MCDM methods $j \in \{S(\text{SAW}), T(\text{TOPSIS}), W(\text{WP})\}$ and three different defuzzification methods $k \in \{C(\text{COA}), G(\text{GMI}), M(\text{MD})\}$ are denoted as DM_{i-j-k} . As an illustration, Table 8 shows the individual ranking outcome of DM_1 .

To measure the consistency degree between the 18 fuzzy group ranking outcomes and the corresponding individual ranking outcomes of the 12 decision makers, Spearman's rank correlation coefficient is used. Table 9 shows the result, where the highest overall consistency degree is 0.353. This result suggests that the group ranking outcome produced by the A-S-C method should be used, as it is most consistent with the views of individual decision makers as a whole, thus most acceptable by them. Accordingly, the most preferred green bus fuel technology for buses in Taiwan is natural gas (A_1).

It is noteworthy that the selection of the group ranking outcome produced by the A-S-C method is justifiable only for the problem data set used in the empirical study. Different problem data sets may result in a different method being selected. This suggests that no single best method can be assumed for the general fuzzy group MCDM problem that requires cardinal ranking. In solving a given fuzzy group MCDM problem with many methods available and acceptable to the decision makers (not necessarily limited to the methods presented in the paper), the method selection approach developed in this paper can be applied to all available methods for identifying the most preferred ranking outcome from the perspective of all decision makers as a whole.

6. Conclusion

There are normally a number of methods available for solving fuzzy group MCDM problems, defined by a given set of fuzzy weight vectors and fuzzy decision matrices. For a given problem, inconsistent ranking outcomes are often produced by different fuzzy MCDM methods. Despite the importance of validating the ranking outcomes produced by different methods, very few studies have been conducted to help a group of decision makers deal with the ranking inconsistency problem produced by different methods. In this

paper, we have developed a new empirical approach for selecting a fuzzy group MCDM method that produces the most preferred group ranking outcome for a given problem data set. We have presented a green bus technology selection problem to illustrate how the approach can be used to help select the most preferred group ranking outcome. With its simplicity in both concept and computation, the approach can be applied in general fuzzy group decision problems solvable by many fuzzy group MCDM methods. It is particularly suited to large-scale fuzzy group MCDM problems where the ranking outcomes produced by different methods differ significantly.

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