An Individual Welfare Maximization Algorithm for Electricity Markets

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***Abstract*—An algorithm that allows a market participant to maximize its individual welfare in electricity spot markets is presented. The use of the algorithm in determining market equilibrium points, called Nash equilibria, is demonstrated. The start of the algorithm is a spot market model that uses the optimal power flow (OPF), with a full representation of the transmission system and inclusion of consumer bidding. The algorithm utilizes price and dispatch sensitivities, available from the Hessian matrix and gradient of the OPF, to help determine an optimal change in an individual's bid. The algorithm is shown to be successful in determining local welfare maxima, and the prospects for scaling the algorithm up to realistically sized systems are very good. Nash equilibria are investigated assuming all participants attempt to maximize their individual welfare. This is done by iteratively solving the individual welfare maximization algorithm until all individuals stop modifying their bids.**

***Index Terms*—Economics, markets, Nash equilibrium, optimal power flow (OPF), power systems, welfare maximization.**

I. INTRODUCTION

ELECTRICITY markets throughout the world are developing that have the structure of a power pool. These pools take bids from market participants and use spot pricing theory to determine the market prices. Examples include Australia [1], Argentina [1], the PJM Interconnection [2], and the New England Power Pool [3]. In PJM, spot prices are called *locational marginal prices (LMPs)*. Participants in these markets need tools that allow them to determine their optimal bidding behavior. This paper develops a new algorithm based on Newton's method for use in maximizing an individual's welfare. This algorithm is shown to be successful on several sample systems, and the prospects of scaling the algorithm up to systems of realistic size appear very good.

Applications of a welfare maximization tool would be varied. For market participants the presence of transmission congestion presents opportunities to sell generation into subdivided markets in which local demand is high and the number of sellers low. Tools are needed to help market participants devise optimal bidding strategies. Conversely, regulators such as U.S. Federal Energy Regulatory Commission (FERC), the U.S. Department of Justice (DOJ), and the state regulatory commissions need to be vigilant against anticompetitive acts by market participants. For example, if a particular entity owns sufficient generation it may be possible to manipulate the market in such a way as to deliberately induce congestion in order to raise prices [4], [5]. The recent wave of mergers and proposed mergers in the U.S. requires that regulators have access to tools to assess the potential for this type of manipulation. FERCs need for such a tool is described in its Order 592 "Policy Statement on Utility Mergers" in December of 1996 [6], and its formal adoption of the Department of Justice/Federal Trade Commission (DOJ/FTC) Horizontal Merger Guidelines [7]. Both the DOJ and the FERC, in a recent proposed rule-making, explicitly stated a desire for computer models [8].

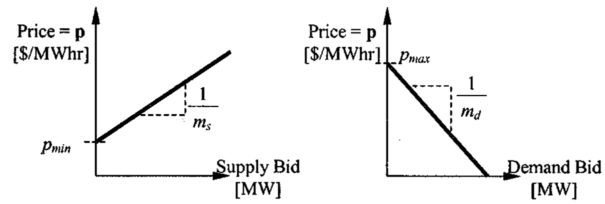

Fig. 1. Consumer and supplier bid curves.

It is also shown that the algorithm can be used to model the behavior of welfare maximizing market individuals. With this ability, game theoretic concepts such as market equilibrium points can be investigated as was done in [9]. Market equilibrium points were found in [9] by iteratively solving the individual welfare maximization problem for each market participant until bids became constant. A similar technique is used in conjunction with the new individual welfare maximization and results are encouraging.

II. ELECTRICITY MARKET SETUP USING OPTIMAL POWER FLOW (OPF)

The setup of the electricity market simulation from [4], [10], [11] is used, and is briefly summarized here for convenience. Market suppliers submit bids that consist of MW outputs, along with associated prices. These supplier bids are increasing functions. Market consumers submit bids that consist of MW demands, along with associated prices. These consumer bids are decreasing functions. Example bid functions for suppliers and consumers are shown in Fig. 1.

For the market simulation, these bids are treated as the marginal cost or benefit of the bidder. The bids are then taken as inputs to an optimal power flow (OPF) that maximizes social welfare to determine supplies, demands, and prices. For this paper, the bids are limited to linear bids as shown in Fig. 1. The Lagrange multipliers of the OPF solution determine the spot prices $\mathbf{p}$. The suppliers in the market are paid the spot price at

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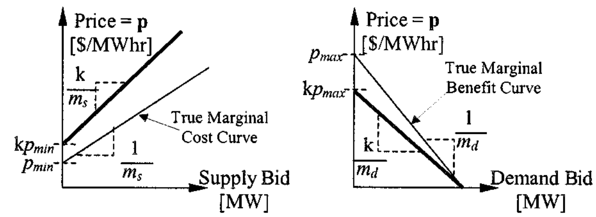

Fig. 2. Bidding variation for supply and demand.

their node, and the consumers are charged the spot price. A thorough treatment of the mathematics involved in integrating bids into the OPF formulation is provided in [10].

III. Individual Player Optimization Problem

Market participants should not be restricted to either a single generator or a single load. Any combination of several generators and loads could constitute an economic entity. Therefore, the word *individual* is defined as a set of supplies and demands whose bidding is controlled by a single entity. This section develops an algorithm for determining a bid that maximizes individual welfare.

For this paper, bids are restricted to linear functions, and the variation of this function is limited to varying a single parameter k for each consumer or supplier as shown in Fig. 2. Parameter k varies the bid from the true marginal curve. The supply bid reflecting true marginal cost is $p(s) = s/m_s + p_{\min}$, while for the consumer, $p(d) = -d/m_d + p_{\max}$ is the true marginal benefit bid.

While this limits market behavior, [11] shows that the shape of this curve is not important to the individual for a single market solution. Note that modifying a bid this way is the same as multiplying the cost or benefit function used in the OPF by k: $C(s) = k(bs + cs^2)$ and $B(d) = k(bd - cd^2)$ [10].

An individual wants to maximize the total welfare of all consumers and suppliers it controls. A consumer's welfare is the amount of benefit received from using the power, minus the expenses incurred in purchasing it. Similarly, a supplier's welfare is defined as the amount of revenue received from selling the power, minus the cost of supplying it

$$\begin{aligned} f(\mathbf{s}, \mathbf{d}, \boldsymbol{\lambda}) &= \sum_{\substack{i=\text{controlled}\\ \text{demands}}} [\underbrace{B_i(d_i)}_{\text{Benefits}} - \underbrace{\lambda_i d_i}_{\text{Expenses}}] \\ &\quad + \sum_{\substack{\text{controlled}\\ \text{supplies}}} [-\underbrace{C_i(s_i)}_{\text{Costs}} + \underbrace{\lambda_i s_i}_{\text{Revenues}}] \\ &= \underbrace{(-\mathbf{d}^T \mathbf{C_d} \mathbf{d} + \mathbf{B_d}^T \mathbf{d})}_{\text{Benefits}} \\ &\quad - \underbrace{(\boldsymbol{\lambda}^T \mathbf{d})}_{\text{Expenses}} - \underbrace{(\mathbf{s}^T \mathbf{C_s} \mathbf{s} + \mathbf{B_s}^T \mathbf{s})}_{\text{Costs}} + \underbrace{(\boldsymbol{\lambda}^T \mathbf{s})}_{\text{Revenues}} \end{aligned} \tag{1}$$

where

$\mathbf{C_d}$ and $\mathbf{C_s}$ diagonal matrices of quadratic coefficients for cost functions and benefit functions;

$\mathbf{B_d}$ and $\mathbf{B_s}$ vectors of linear coefficients for supply cost functions and demand benefit functions, respectively.

Note $f(\mathbf{s}, \mathbf{d}, \boldsymbol{\lambda})$ is not an explicit function of its bid variable $\mathbf{k}$. However, f is an implicit function of $\mathbf{k}$ since $\mathbf{s}$, $\mathbf{d}$, and $\boldsymbol{\lambda}$ are all determined by an OPF solution which is a function of $\mathbf{k}$.

Assuming the individual has some estimate of what other market individuals are going to bid, the individual's goal is to maximize its welfare by choosing a bid which is the best response to the other individuals' bids. As a result, the maximization of an individual's welfare forms a nested optimization problem where the individual maximizes its welfare subject to an OPF solution which maximizes social welfare based on all bids in the market

$$\begin{aligned} &\max_{\mathbf{k}} \quad f(\mathbf{s}, \mathbf{d}, \lambda) \\ &\text{s.t.} \quad (\mathbf{s}, \mathbf{d}, \lambda) \text{ are determined by} \\ &\qquad \begin{pmatrix} \max\limits_{\mathbf{x},\mathbf{s},\mathbf{d}} & B_{ind}(\mathbf{d}, \mathbf{k}) - C_{ind}(\mathbf{s}, \mathbf{k}) \\ & +B_{comp}(\mathbf{d}) - C_{comp}(\mathbf{s}) \\ \text{s.t.} & \mathbf{h}(\mathbf{x}, \mathbf{s}, \mathbf{d}) = \mathbf{0} \\ & \mathbf{g}(\mathbf{x}, \mathbf{s}, \mathbf{d}) \le \mathbf{0} \end{pmatrix} \end{aligned} \tag{2}$$

where $B_{ind}(\mathbf{d}, \mathbf{k})$ and $C_{ind}(\mathbf{s}, \mathbf{k})$ are the benefit and cost functions of the consumers and suppliers that the individual controls; $B_{comp}(\mathbf{d})$ and $C_{comp}(\mathbf{s})$ are the estimates of the benefit and cost functions that the individual's competitors will submit as bids; $\mathbf{x}$ is the state vector including system voltages and angles; $\mathbf{h}(\mathbf{x}, \mathbf{s}, \mathbf{d})$ are equality constraints such as the power flow equations; and $\mathbf{g}(\mathbf{x}, \mathbf{s}, \mathbf{d})$ are inequality constraints such as line flow limits. Thus, the total societal benefit used by the OPF market model is $B(\mathbf{d}) = B_{ind}(\mathbf{d}, \mathbf{k}) + B_{comp}(\mathbf{d})$, and the total societal cost used is $C(\mathbf{s}, \mathbf{k}) = C_{ind}(\mathbf{s}, \mathbf{k}) + C_{comp}(\mathbf{s})$.

IV. Solution Method by Iterative Means

An iterative approach is proposed for solving (2) to determine $\mathbf{k}$ [11]. The following is a brief summary of this.

```
Algorithm: Preliminary Individual Welfare
Maximization
1.  Choose an initial guess for vector k.
2.  Solve the OPF maximization of social
  welfare given the individual's assump-
  tion of other individual's bids and the
  individual's guess at its own vector k.
3.  Use (3) to determine a step direction
  for vector k.
4.  If ||k_new - k_old|| is below some tolerance,
  then stop; else go back to step 2.
```

Thus, the algorithm begins with an initial guess of $\mathbf{k}$. Next, the OPF problem is solved assuming the specified $\mathbf{k}$. Then, from the information available at this OPF solution, the individual's profit sensitivity to variations in its bid can be used to determine a Newton-step that improves profits. This Newton-step is defined the customary way as

$$\mathbf{k}_{new} = \mathbf{k}_{old} - \left[\frac{\partial^2 f}{\partial \mathbf{k}^2}\right]^{-1}\Bigg|_{\mathbf{k}_{old}} \frac{\partial f}{\partial \mathbf{k}}\Bigg|_{\mathbf{k}_{old}}. \tag{3}$$

The evaluation of (3) requires determining $\partial^2 f/\partial \mathbf{k}^2$ and $\partial f/\partial \mathbf{k}$. These can be shown to be functions of the Hessian and

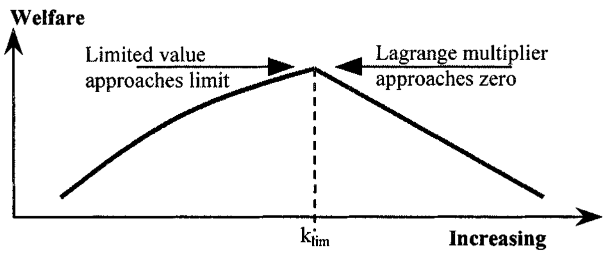

Fig. 3. Binding inequality change in one dimension.

Lagrange function at the solution of the OPF. This derivation is fully covered in [11] and the computational requirements for calculating $\partial^2 f/\partial \mathbf{k}^2$ and $\partial f/\partial \mathbf{k}$ are shown to be very small.

The *Preliminary Individual Welfare Maximization Algorithm* is effective as long as the binding inequalities of the OPF algorithm do not change. Changes in binding inequalities result in discontinuities of $\partial f/\partial \mathbf{k}$, which means that the function f becomes nondifferentiable. A change in binding inequality, however, can be detected from other available information. From one side of the nondifferentiable point, the value limited by the inequality approaches its limit. From the other side, the Lagrange multiplier approaches zero. This is shown in Fig. 3.

If only one bid parameter and one inequality constraint are being considered, the *Preliminary Individual Welfare Maximization Algorithm* could be simply modified so a multiplier reduces the step direction determined by (3) if this step direction will move across a nondifferentiable point. The multiplier would then bring the answer directly to this nondifferentiable point [11].

In order to extend this idea to the more general case of multiple bid parameters and binding inequality constraints, consider the origins of the Newton step described in (3). Newton's method solves for a function's zero crossing by approximating it as a linear function using its present value and derivative. In a maximization problem, the zero crossing of the *derivative* of the objective function is desired; therefore, the objective function is inherently modeled as a second-order Taylor series

$$\begin{aligned} f(\mathbf{k}_{new}) &= f(\mathbf{k}_{old} + \mathbf{\Delta k}) \\ &\approx f(\mathbf{k}_{old}) + (\mathbf{\Delta k})^T \left.\frac{\partial f}{\partial \mathbf{k}}\right|_{\mathbf{k}_{old}} \\ &\quad + \frac{1}{2}(\mathbf{\Delta k})^T \left.\frac{\partial^2 f}{\partial \mathbf{k}^2}\right|_{\mathbf{k}_{old}} (\mathbf{\Delta k}). \end{aligned} \tag{4}$$

Equation (3) is then derived by solving $\partial f(\mathbf{k}_{old} + \mathbf{\Delta k})/\partial \mathbf{\Delta k} = \mathbf{0}$.

In order to follow the analogy of Fig. 3, the $\mathbf{\Delta k}$ that maximizes (4) without crossing any constraint boundaries is desired. This is determined by solving (5)

$$\begin{aligned} \max_{\mathbf{k}} \quad & f(\mathbf{k}_{old}) + (\mathbf{\Delta k})^T \left.\frac{\partial f}{\partial \mathbf{k}}\right|_{\mathbf{k}_{old}} + \frac{1}{2}(\mathbf{\Delta k})^T \frac{\partial^2 f}{\partial \mathbf{k}^2}(\mathbf{\Delta k}) \\ \text{s.t.} \quad & (\mathbf{\Delta k})^T \frac{\partial g_m}{\partial \mathbf{k}} + g_m \le 0 \qquad \forall g_m \le 0 \\ & -(\mathbf{\Delta k})^T \frac{\partial \lambda_{gm}}{\partial \mathbf{k}} - \lambda_{gm} \le 0 \qquad \forall\, g_m = 0. \end{aligned} \tag{5}$$

Note that $\partial^2 f/\partial \mathbf{k}^2$ is negative definite. Furthermore, the only derivative in (5) that is not discussed in [11] is $\partial g_m/\partial \mathbf{k}$. This derivative can be readily calculated using the chain rule

$$\frac{\partial g_m}{\partial \mathbf{k}} = \frac{\partial \mathbf{d}}{\partial \mathbf{k}}\frac{\partial g_m}{\partial \mathbf{d}} + \frac{\partial \mathbf{s}}{\partial \mathbf{k}}\frac{\partial g_m}{\partial \mathbf{s}} + \frac{\partial \mathbf{x}}{\partial \mathbf{k}}\frac{\partial g_m}{\partial \mathbf{x}}. \tag{6}$$

While adding another maximization problem as an outer loop to the problem may seem difficult, it is not because (5) is a very simple constrained maximization problem. It is a quadratic objective function with linear inequality constraints: a quadratic programming problem. Many very efficient methods for the solution of this problem exist [12] and can be used to quickly solve the problem described by (5) in a time much faster than the solution of the OPF inner loop. Thus, solution time will be largely dependent on the number of OPF iterations needed. With this further development, the new algorithm is proposed.

```
Algorithm: Individual Welfare Maximization
1.  Choose an initial guess for vector k.
2.  Solve the OPF maximization of social
    welfare given the individual's assump-
    tion of other individuals' bids and the
    individual's guess at its own vector k.
3.  Use (5) to determine a step direction
    for vector k.
4.  If ||k_new - k_old|| is below some toler-
    ance, then stop; else go back to step 2.
```

Examples demonstrating the use of the *Individual Welfare Maximization Algorithm* are presented throughout the following sections in conjunction with the use of this algorithm in finding economic equilibrium points.

V. Finding a Nash Equilibrium

While the *Individual Welfare Maximization Algorithm* is of use to market participants, using the algorithm as a model of individual behavior enables the study of other interesting market behavior. For example, the determination of economic equilibrium points such as Nash equilibria [13] is of interest.

Definition: Nash Equilibrium:

1) An individual looks at its opponents' behaviors.
2) The individual determines that its best response to its opponents' behaviors is to continue its present behavior.
3) This is true FOR ALL individuals in the market.

To determine a Nash equilibrium the *Individual Welfare Maximization Algorithm* can be iteratively solved by all individuals until a point is reached where each individual's best response is to continue with the same vector of bids. A similar iterative technique for finding Nash equilibria was used in [9], although a very different individual maximization algorithm was used. The following algorithm describes this process.

```
Algorithm: Find Nash Equilibrium
1) Start all individuals with a bid vector
   k = 1.
```

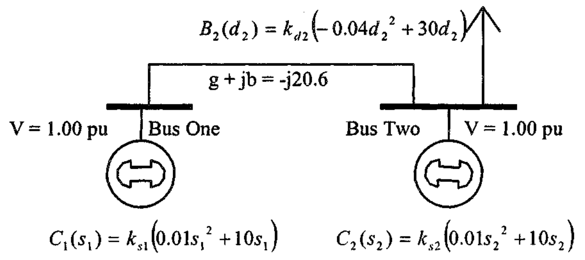

Fig. 4. Two-bus system: Two suppliers and a consumer.

2) Run the *Individual Welfare Maximization Algorithm* for each individual. Update all bids.
3) Continue running this until all individuals stop changing their bids.

A. Two-Supplier Competition With and Without Constraints

To demonstrate the *Find Nash Equilibrium Algorithm*, in [11] the 2-bus example with two suppliers and one consumer shown in Fig. 4 was considered.

Only supplier bidding behavior for this example was considered, therefore it assumed the consumer always bids according to its true benefit function, i.e., $k_{d2} = 1.00$. Maintaining the price-dependent demand is important. Otherwise when a limit is added to the system, supplier 2 could have part of the constant load to serve with no competition. The solution for supplier 2 would be to bid k_{s2} equal to infinity, an unreasonable result.

The results in [11] showed that with no transmission constraint, a Nash equilibrium with both suppliers bidding $k_{s1} = k_s = 1.1502$ existed. It was also shown that this was the only Nash equilibrium.

When adding in an 80–MVA transmission line constraint however, it was shown that no pure strategy Nash equilibrium existed. This was caused by supplier 2 having a nonconvex welfare function that had two local maxima. These results will be compared to supplier versus consumer competition next.

B. Supplier Versus Consumer Competition With No Constraints

Now consider the 2-bus example shown in Fig. 4 again, but assume supplier 2 is removed, and only supplier 1 and the consumer compete by varying their bids. The Nash equilibrium results in bids of $k_{s1} = 1.5714$ and $k_{d2} = 0.8571$. As with two suppliers competing, without the transmission line constraint included the Nash equilibrium is found to be a pure strategy. The algorithm progresses smoothly to its equilibrium. Fig. 5 shows a complete solution to the problem with the optimal response of each participant to any possible bid by the other. The point where the two curves in Fig. 5 meet is the Nash equilibrium point.

C. Generator and Demand Competition With Constraints

Again, it is instructive to look at the same example, but with the addition of a transmission constraint. Analysis is done with the consumer and supplier 1 competing against one another while an 80–MVA transmission line limit is enforced. As with

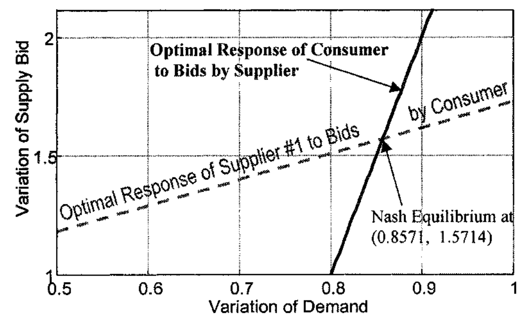

Fig. 5. Supplier's and consumer's optimal responses with no limits.

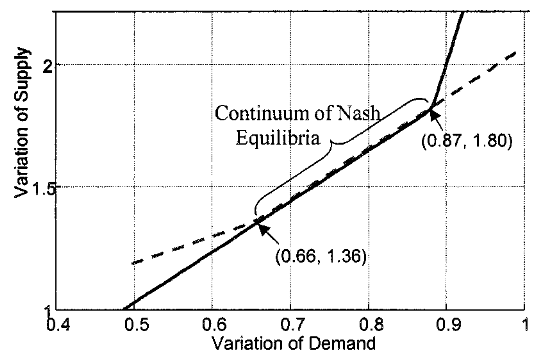

Fig. 6. Supplier's and consumer's optimal responses with 80–MVA limit.

TABLE I
VARIATION OF THE SOLUTION ALONG THE NASH EQUILIBRIUM CONTINUUM

k_{d2}	k_{s1}	Line Flow [MVA]	Price at Nodes 1-2 [$/MWh]	Supply/ Demand [MW]	Consumer Surplus [$/h]	Supplier Profit [$/h]	Cons. Surp + Supp. Prof [$/h]
1.36	0.66	80.0	15.69	77.79	382.3	870.9	1253.2
1.44	0.70	80.0	16.69	77.79	460.0	793.3	1253.2
1.53	0.74	80.0	17.69	77.79	537.6	715.6	1253.2
1.62	0.79	80.0	18.69	77.79	615.3	637.9	1253.2
1.70	0.83	80.0	19.69	77.79	693.0	560.2	1253.2
1.79	0.87	80.0	20.69	77.79	770.7	482.5	1253.2

two supplier competition with a constraint, no equilibrium is reached using the *Find Nash Equilibrium Algorithm*. To better show why no equilibrium point is reached, the optimal response curves over all possible bids by each individual are determined. These are shown in Fig. 6.

No equilibrium is reached by the *Find Nash Equilibrium Algorithm* because the algorithm is bouncing back and forth across a continuum of Nash equilibria. As shown in Fig. 6, a line of equilibrium points exists between $k_{d2} = 0.66/k_{s1} = 1.36$ and $k_{d2} = 0.87/k_{s1} = 1.80$. Table I shows the variation of the market solution along the Nash equilibrium continuum.

To understand what is occurring, consider the case when both supplier and consumer submit bids according to their true benefit and cost functions, i.e., $k_{s1} = 1.00$ and $k_{d2} = 1.00$. The market solution for these bids is a supplier node 1 price of 11.56 $/MWh and a consumer node 2 price of 23.33 $/MWh, while 77.79 MW are exchanged between them. The difference in nodal prices results in a transmission rent collected by the pool operator of (77.79 MW)$*$(23.33 $-$ 11.56 $/MWh) $=$ 907.1 $/h. Thus, a huge amount of money is being wasted as transmission rent due to the transmission line constraint.

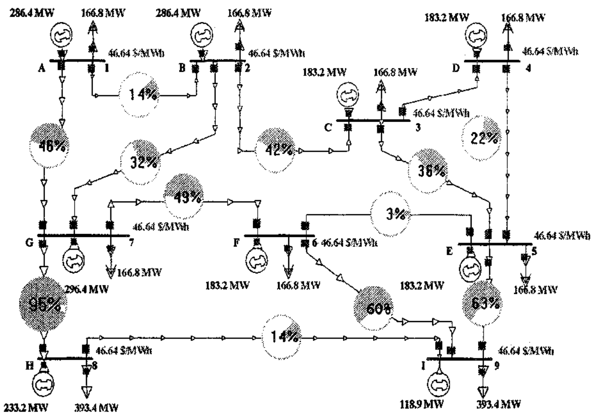

Fig. 7. Nine-bus electricity market.

An intelligent supplier and consumer would find a way to mitigate this expense and come up with a manner in which this transmission rent could be split between the two parties instead of sending it to the pool operator. The profit, surplus, and transmission rent are summarized as follows:

1) consumer surplus: 285.6 \$/h;
2) supplier profit: 60.5 \$/h;
3) transmission rent: 907.1 \$/h;
4) total: 1253.2\$/h.

Note that the total of consumer surplus, supplier profit, and transmission rent is the same as the sum of supplier profit and consumer surplus along the Nash equilibrium continuum (see Table I). Thus, the continuum of Nash equilibrium shown in Fig. 6 represents all the different ways the consumer and supplier can submit bids in a manner which results in a dispatch exactly at the transmission line limit, thus avoiding any transmission rent penalty due to a difference in nodal prices. The continuum of Nash equilibria represents all the different ways to split the transmission rent between the consumer and supplier. The continuum of Nash equilibria represents the ability of a competitive market to determine the best way to utilize scarce transmission resources.

D. Example 9-Bus System Illustrating Market Power

As mentioned in the introduction, the use of computer models for helping illustrate market power is of exceptional interest to the U.S. Department of Justice as well as state regulatory commissions throughout the U.S. An example of the *Find Nash Equilibrium Algorithm* used in the role is provided here.

Consider the 9-bus electricity system shown in Fig. 7 with a supplier and a consumer at each bus. All transmission lines have the same characteristics ($r + jx = 0 + j0.1$; and $C = 0$), and the actual cost curves and benefit curves for the suppliers and consumers are of the form consumer benefit $= B_i(d_i) = b_i d_i + c_i d_i^2$ and supplier cost $= C_i(s_i) = b_i s_i + c_i s_i^2$ with the coefficients b and c, as shown in Table II.

As a reference point, the bids corresponding to true marginal benefit and welfare from all consumers and suppliers are assumed, and the OPF that maximizes social welfare is solved. The results are those shown in Fig. 7 with a market price of \$46.64/MWh throughout the system. Suppliers at buses 7 and 8 have a combined profit of \$4394.00/h + \$5439.29/h = \$9833.29/h. Also note that the flow on the transmission line from bus 7 to bus 8 is 189.5 VA.

Now, assume that all the consumers in this market are fringe participants and exercise no strategic bidding behavior, i.e., they always submit offers representative of their true marginal benefit curve. Assume that all the suppliers, however, do exhibit strategic bidding behavior and will modify their bids in hopes of increasing their individual welfare.

Now suppose suppliers 7 and 8 collude with the hopes of increasing their combined profits. Both could raise their prices slightly hoping to increase profit. Looking at Fig. 7, however, one notices that the line between buses 7 and 8 is loaded at 95% of its limit. As a result, suppliers 7 and 8 might also consider colluding to overload this line, thus increasing the price which supplier 8 will receive for its power. To do this, supplier 7 will have to lower its price and reduce its profit with the understanding that supplier 8 can increase its price and profit because of the overload. In order to force the *Individual Welfare Maximization Algorithm* into the region of parameter space which contains the other anticipated local maximum, the bid of supplier 8 is set to $k_8 = 2.0$. Then, the algorithm is run again resulting in convergence to another local maximum.

Both of these scenarios are considered, and the *Individual Welfare Maximization Algorithm* is solved for each supplier (with suppliers 7 and 8 acting together) until Nash equilibria are reached. Different Nash equilibria are found for each scenario. Each supplier is unable to raise its profit by either lowering or raising its bid at the equilibrium points.

The results at the Nash equilibrium reached when suppliers 7 and 8 both try to raise prices while suppliers 1–6 and 9 individually try to maximize welfare are shown in Table III.

Note that suppliers at buses 7 and 8 have a combined profit of \$4824.89/h + \$5813.56/h = \$10 638.45/h. Also note that the flow on the transmission line from bus 7 to bus 8 is 190.0 MVA. In this scenario the prices increase to \$48.51/MWh. This is in increase of 4.0% above the \$46.64/MWh seen when all supplier bid their marginal cost.

Results for the Nash equilibrium when suppliers 7 and 8 collude to try and overload the transmission line between buses 7 and 8 are shown in Table IV.

Note that suppliers at buses 7 and 8 have a combined profit of \$4397.38/h + \$6979.01/h = \$11 376.39/h. The flow on the transmission line from bus 7 to bus 8 is at its limit of 200 MVA. Comparing Tables III and IV, suppliers 7 and 8 are able to

TABLE II
COST AND BENEFIT EQUATION COEFFICIENT FOR ILLUSTRATIVE EXAMPLE

Bus	Supplier b Coefficient	Supplier c Coefficient	Consumer b Coefficient	Consumer c Coefficient
1 (A)	18	0.05	80	-0.10
2 (B)	18	0.05	80	-0.10
3 (C)	21	0.07	80	-0.10
4 (D)	21	0.07	80	-0.10
5 (E)	21	0.07	80	-0.10
6 (F)	21	0.07	80	-0.10
7 (G)	17	0.05	80	-0.10
8 (H)	0	0.10	440	-0.50
9 (I)	30	0.07	440	-0.50

TABLE III
NASH EQUILIBRIUM RESULTS WHEN SUPPLIERS 7 AND 8 BOTH RAISE PRICES

Bus	Price [$/MWh]	Supplier Gen [MW]	Supplier Profit [$/h]	Consumer Dem [MW]	Consumer Surplus [$/h]
1	48.51	275.8	4,612.36	157.4	2,478.55
2	48.51	275.8	4,612.36	157.4	2,478.55
3	48.51	183.0	2,690.69	157.4	2,478.55
4	48.51	183.0	2,690.69	157.4	2,478.55
5	48.51	183.0	2,690.69	157.4	2,478.55
6	48.51	183.0	2,690.69	157.4	2,478.55
7	48.51	262.1	4,824.89	157.4	2,478.55
8	48.51	216.1	5,813.56	391.5	76,630.97
9	48.51	123.1	1,218.26	391.5	76,630.97
Totals		**1885.0**	**31,844.19**	**1885.0**	**170,611.81**

TABLE IV
NASH EQUILIBRIUM WHEN SUPPLIERS 7 AND 8 TRY TO OVERLOAD A LINE

Bus	Price [$/MWh]	Supplier Gen [MW]	Supplier Profit [$/h]	Consumer Dem [MW]	Consumer Surplus [$/h]
1	47.20	242.92	4,143.22	163.99	2,689.30
2	47.75	256.96	4,341.99	161.27	2,600.88
3	49.38	188.48	2,861.81	153.11	2,344.38
4	49.92	191.05	2,970.40	150.39	2,261.83
5	50.47	192.08	3,077.19	147.67	2,180.65
6	49.92	191.03	2,970.73	150.38	2,261.52
7	46.66	298.83	4,397.38	166.72	2,779.42
8	57.55	173.68	6,979.01	382.45	73,133.35
9	52.66	128.31	1,755.26	387.34	75,015.49
Totals		**1,863.33**	**33,497.01**	**1863.33**	**165,266.80**

increase their combined equilibrium profit from $10 638/h to $11 376/h, an increase of 6.9%, when they choose a strategy of overloading the transmission line. In doing so they increase the equilibrium prices at buses 8 and 9 to $57.55/MWh and $52.66/MWh, which are 23.4% and 12.9% above the social welfare solution price. Thus, there is some concern regarding localized market power if suppliers 7 and 8 are to merge. If one ignores the transmission system, however, and only considers the problem of suppliers 7 and 8 acting to raise prices together, these market power concerns would not be as readily apparent.

In order to get a better grasp of what the *Individual Welfare Maximization Algorithm* is facing, a complete solution to the problem is performed when all other suppliers bid $k = 1.0$. The bid for supplier 7 is then varied between 0.8 and 1.5 while varying the bid for supplier 8 between 1.0 and 1.8. Fig. 8 shows a contour plot of the combined welfare in this region.

As can be seen by Fig. 8, there are indeed two local maxima for the problem separated by a constraint boundary. This constraint boundary describes the region of the parameter space which results in the line between node 7 and 8 being exactly at its limit of 200 MVA. This example shows that the individual welfare function even for very simple systems results in problems with many local optima. This continued reminder motivates the investigation of other more globally oriented optimization algorithms. While the calculus-based algorithm will be useful in finding a local maxima, it will invariably have difficulties when trying to find a general global maximum.

It is of some use, however, because the local maxima which result are due to the physical constraints of the system. The constraints in the system that the individual has the ability to manipulate will likely be known due to the individual's experience.

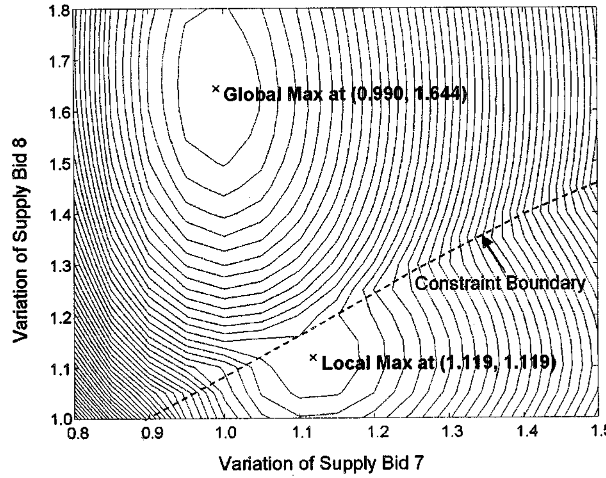

Fig. 8. Contour plots of combined profit of supplier 7 and 8.

Because of this, the algorithm user could push the bids found for one local optimum into a region of the bidding space that would converge to another anticipated local optimum. Indeed, this is exactly what was done in the previous 9-bus example to find the second local maximum. Because of this, the calculus-based method will be of some use even without using a more globally oriented optimization routine. A global optimization technique, such as a genetic algorithm (GA), would be useful in finding bidding strategies that an individual's experience does not point them toward.

VI. CONCLUSIONS

The *Individual Welfare Maximization Algorithm* presented will be of great use to individual market participants for market analysis. However, others, such as industry regulators, are interested in studying market equilibrium behavior. Using the *Individual Welfare Maximization Algorithm* as a model of individual bidding behavior, the entire market can be simulated with individuals modifying their bids with the objective of maximizing welfare. This was done iteratively until equilibrium points, called Nash equilibria, were reached. This technique was shown to be very useful for finding Nash equilibria, as long as they existed. It also highlighted the fact that Nash equilibria do not always exist, and that when they do exist they may not be unique.

The new *Individual Welfare Maximization Algorithm* technique developed is a calculus-based optimization routine. However, it was shown that the individual welfare, even for simple systems, can be a highly nonconcave function resulting in many local optima. The local optima correspond to the ability of the individual to manipulate its bidding strategy to take advantage of a system constraint, such as a transmission line constraint. Because these local optima will correspond to physically understandable phenomena, it is hoped that the user of the *Individual Welfare Maximization Algorithm* will be able to nudge the initial guesses of the algorithm into regions that will correspond to these local optima. This will make the algorithm of use on its own; however, a more global optimization routine such as a GA is also being investigated.

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